**On Energy-Momentum Tensors of Gravitational Field.**

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Summary

Several energy-momentum "tensors" of gravitational field are considered and compared in the lowest approximation. Each of them together with energy-momentum tensor of point-like particles satisfies the conservation laws when equations of motion of particles are the same as in general relativity. It is shown that in Newtonian approximation the considered tensors differ one from the other in the way their energy density is distributed between energy density of interaction (nonzero only at locations of particles) and energy density of gravitational field.

Starting from Lorentz invariance the Lagrangians for spin-2, mass-0 field are considered. They differ only by divergences. From these Lagrangians by Belinfante-Rosenfeld procedure the energy-momentum tensors are build. Using each of these tensors in 3-graviton vertex we obtain the corresponding metric of a Newtonian center in $G^2$ approximation. Only one of these "field-theretical"tensors (namely the half sum of Thirring tensor and tensor obtained from Lagrangian given by Misner, Thorne and Wheeler) leads to correct value of the perihelion shift. This tensor does not coincide with Weinberg's one (directly obtainable from Einstein equation) and gives metric of a spherical body differing (in space part of metric in the first nonlinear approximation) from Schwarzschild field in harmonic coordinates. As a result a *relativistic* particle in such field must move note according general relativity prescriptions.

This approach puts the gravitational energy-momentum tensor on the same footing as any other energy-momentum tensor.

# 1 Introduction

General relativity is a complete, elegant, and self-consistent theory. Yet there is a necessity to obtain gravity by field-theoretical means starting from flat spacetime, see e.g.[1-4]. It is widely believed that on this way even dropping the general covariance requirement we naturally get general relativity. It is supposed that in the lowest nonlinear approximation this is demonstrated in detail by Thirring [2]. Yet this conclusion can not be drawn from [2], see Sec.2.

The energy-momentum tensor of material fields in general relativity is obtained from that without gravitational field by equivalence principal (comma goes to semicolon). This means that general covariance dictates the form of vertices containing material fields. Even in this case other considerations may lead

to modifications. So conformal invariance leads to Chernikov-Tagirov energy-momentum tensor [5]. Dropping general covariance gives more freedom in choosing vertices.

Since the gravitational collapse is considered as the greatest crisis in physics [6] the research into possible alternative theories acquire especial significance. It is quite natural to make the first step and to consider the simplest processes by utilizing vertices; the graviton propagator is known by analogy with electrodynamics.

In the lowest nonlinear approximation it is necessary to know only 3-graviton vertex. We assume the simplest possibility: the source of graviton is the energy-momentum tensor of two other gravitons. In higher approximations probably other vertices will be needed. Along this path one can find out what theories are possible without assuming general covariance and a priori restriction on vertices. An important step in this direction was made by Thirring [1-2]. We continue his investigation in the same approximation and restrict ourselves to point-like classical particles as sources of gravitation. Mainly we are interested in the simplest system consisting of a Newtonian center and test particle moving in its field.

In general relativity classical particles move along geodesics in Riemannian space. This is the incarnation of equivalence principle and it is more reliable than specific equation determining the gravitational field [9]. As to the equation determining gravitational field, it is possible to think that the phenomenological field-theoretical approach will lead to more complicated algorithm for getting the field. An interesting possibility in this direction was pointed out by Schwinger [10].

It is reasonable to believe together with Einstein that for some reason or other the singular behaviour near the gravitational radius does not correspond to reality, see §15 in .Pais‘s book [11] and Einstein paper [12]. . At present the Schwarzschild singularity is considered as fictitious by many researchers because the geometry is nonsingular there. See however the text after (2.2.6) in [13] and after eq.(9.40) in [14], where they say convincingly about *physical* singularity. By field-theoretical approach it is difficult to understand why in a finite system the acceleration of a freely falling particle becomes unlimited when it nears the horizon. Such a behaviour should be connected with the fact that according to [15] the gravitational energy in vacuum outside the sphere of radius R goes to $-\infty$ for $R \to r_g$. In conformity with this the energy of matter and gravitational field inside the sphere of radius $R$ goes to $+\infty$, in such away that total energy of spherical body is equal to its mass. But if a theory predicts that the absolute value of field energy outside sphere of radius R might be greater than total energy of a body then the analogy with electrodynamics suggests that the concept of external field becomes inapplicable [16]. The belief in general relativity in similar circumstances is based upon the concept of nonlocalazability of gravitational energy, see, e.g. §20.4 in [6]. What is more, general relativity does not need as a rule the gravitational energy -momentum pseudotensor.

The situation changes drastically, when we begin to construct gravity theory starting from flat spacetime and assume that in 3-graviton vertex each graviton interacts with energy-momentum tensor formed by two other graviton. Then the nonlinear correction to the motion of a test particle depends on the chosen energy-momentum tensor. The latter is build from field Lagrangian, which is not unique as one can add to it some divergence terms. This leads to different energy-momentum tensors. They can give rise to gravitational energy densities, which may have even different signs. The question of sign of energy density is of interest by itself. Provided the sign turns out to be positive, one should expect the weakening of gravitational interaction at $r \sim r_g = 2GM$ in comparison with Newtonian one in order that the gravitational energy outside the sphere of radius $r$ were much less than the mass $M$ of the center. The possibility of decreasing of interaction at small distances is suggested also by the behaviour of attraction force between two bodies supported by Weyl's strut, see, §35 [17].

In order to understand in what way the various energy-momentum tensors differ one from the other we consider the following tensors: Thirring's [1,2], Landau-Lifshitz's [16], Papapetrou-Weinberg's [9] and tensor obtained from the Lagrangian given in Exercise 7.3 in [6]. The second and third tensors are representatives of general relativity, the rest are build from Lagrangians of free field of spin-2, mass-0 particles and symmetrized by the Belinfante-Rosenfeld procedure, see, §1 Ch.7 in [18].

In the considered approach it is suitable to subdivide the 3-graviton vertex into three vertices in accordance with three possibilities for choosing two gravitons out of three to form the energy-momentum tensor,- the source for the third graviton, see Sec.2. So three diagrams contribute to nonlinear correction to the field. In contrast with this the energy-momentum tensor, figuring in the solution of Einstein equation by iteration procedure, is so defined that the correction to field is given simply by means of propagator, i.e. by one diagram only.

The main result of the paper is this: starting from quadratic Lagrangians (differing by divergence terms) of spin-2, mass-0 particles, the energy-momentum tensors are constructed by Belinfante-Rosenfeld procedure. It turns out that only certain combination of these tensors (used in 3-graviton vertex) is fitted for correct description of perihelion shift. This combination does not coincide with Papapetrou-Weinberg tensor.

The investigation of possibilities of phenomenological approach to gravitation without use of general covariance seems to us very promising. Valuable undertaking in this direction was made in [19].

## 2 Thirring's energy-momentum tensor

Throughout the paper we use

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1}$$

In Sections 2, 3, 5 $\eta_{\mu\nu} = diag(1,-1,-1,-1,)$; in Sections 4, 6 and Appedix $\eta_{\mu\nu} = diag(-1,1,1,1,)$. In this Sec. we use Thirring‘s notation [2]; both greek and latin indices run from 0 to 3.

The gravitational field is described by the symmetric tensor $h_{\mu\nu}$, which contains spin-2 and lower spins, see, e.g. [3]. The unnecessary spins (spin-1 and one of spin-0) are excluded by Hilbert gauge :

$$\bar{h}^{\mu\nu}{}_{,\nu} \equiv (h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h)_{,\nu} = 0, \quad h = h^{\sigma}{}_{\sigma}, \quad h_{,\nu} \equiv \frac{\partial}{\partial x^{\nu}}h. \tag{2}$$

One way to obtain Thirring‘s tensor is to start from general relativity Lagrangian $\sqrt{-g}R$. If we remove terms with second derivatives of $g_{\mu\nu}$ into divergence terms and drop the latter, we get the function $G(x)$ in eq.(93.1) in [16]. Retaining in it only quadratic in $h_{\mu\nu}$ terms we get

$$G(x) = \frac{1}{4}[h_{\mu\nu,\lambda}h^{\mu\nu,\lambda} - 2h_{\mu\nu,\lambda}h^{\lambda\nu,\mu} + 2h_{\nu\mu}{}^{,\mu}h^{,\nu} - h_{,\lambda}h^{,\lambda}] \tag{3}$$

This is equivalent to Thirring‘s Lagrangian [2]

$$\overset{f}{L} = \frac{1}{2}[\psi_{\mu\nu,\lambda}\psi^{\mu\nu,\lambda} - 2\psi_{\mu\nu,\lambda}\psi^{\lambda\nu,\mu} + 2\psi_{\mu\nu}{}^{,\mu}\psi^{,\nu} - \psi_{,\lambda}\psi^{,\lambda}]. \tag{4}$$

Here

$$\psi_{\mu\nu} = -h_{\mu\nu}/2f, \quad \psi = \psi_{\sigma}{}^{\sigma}, \quad f^2 = 8\pi G, \quad G = 6.67 \cdot 10^{-8} cm^3/g \cdot sec^{2.} \tag{5}$$

Using $\psi_{\mu\nu}$ instead of $h_{\mu\nu}$ is justified because then the analogy with electrodynamics becomes more close: $\psi_{\mu\nu}$ is analogous to vector- potential $A_{\mu}$ and has the same dimensionality, $M\sqrt{G}$ has the dimensionality of electromagnetic charge. We note that the Lagrangian (4) exactly corresponds to Schwinger‘s Lagrangian [10], who uses the notation $\eta_{\mu\nu} = diag(-1,1,1,1,)$ and $2h^{Sch}_{\mu\nu} = -h^{T}_{\mu\nu} = -h_{\mu\nu}$.

The canonical energy-momentum tensor following from (4), has the form

$$\overset{f}{T}{}^{\gamma\delta} = \varphi^{\mu\nu,\delta}\varphi_{\mu\nu}{}^{,\gamma} - \frac{1}{2}\varphi^{,\delta}\varphi^{,\gamma} - 2\varphi^{\mu\nu,\delta}\varphi^{\gamma}{}_{\nu,\mu} - \eta^{\gamma\delta}\overset{f}{L},$$
$$\overset{f}{L} = \frac{1}{2}[\varphi_{\mu\nu,\lambda}\varphi^{\mu\nu,\lambda} - \frac{1}{2}\varphi_{,\lambda}\varphi^{,\lambda} - 2\varphi_{\mu\nu,\lambda}\varphi^{\lambda\nu,\mu}], \tag{6}$$

$$\varphi_{\mu\nu} \equiv \bar{\psi}_{\mu\nu} = \psi_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\psi, \quad \psi = \psi_{\sigma}{}^{\sigma}. \tag{7}$$

Using $\varphi_{\mu\nu}$ instead of $\psi_{\mu\nu}$ is handy as many expressions become more compact and the consequences of imposition of Hilbert gauge more clear.

The energy-momentum tensor for a static point-like mass (Newtonian center)

$$\overset{M}{T}_{\mu\nu} = M\delta(\vec{x})\delta_{\mu 0}\delta_{\nu 0}. \tag{8}$$

In linear approximation this source generate the field

$$\varphi_{\mu\nu} = -\bar{h}_{\mu\nu}/2f = \frac{fM}{4\pi|\vec{x}|}\delta_{\mu 0}\delta_{\nu 0}, \quad \bar{h}_{\mu\nu} = 4\phi\delta_{\mu o}\delta_{\nu 0}, \tag{9}$$

satisfying Hilbert condition (2). For one Newtonian center $\phi = -GM/r$. For several centers

$$\phi = -G\sum_a \frac{m_a}{|\vec{r} - \vec{r}_a|}. \tag{10}$$

In terms of

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}, \quad \bar{h} = \bar{h_\sigma}^{\sigma}, \tag{11}$$

we have

$$h_{\mu\nu} = 2\phi\delta_{\mu\nu}, \quad h = {h_\sigma}^{\sigma} = -4\phi = -\bar{h}. \tag{12}$$

The energy density of field (9) is positive

$$\overset{f}{T}{}^{00} = \frac{1}{8\pi G}(\nabla\phi)^2. \tag{13}$$

But $\overset{f}{T}{}^{\gamma\delta}$ ought to be supplemented to symmetric one by the spin part:

$$\theta^{\gamma\delta} = \overset{f}{T}{}^{\gamma\delta} + \overset{s}{T}{}^{\gamma\delta}. \tag{14}$$

For Newtonian center Thirring obtains

$$\overset{s}{T}{}^{\gamma\delta} = -\frac{1}{\pi G}(\nabla\phi)^2\delta_{\gamma 0}\delta_{\delta 0}, \quad \overset{s}{\bar{T}}{}^{\gamma\delta} = -\frac{1}{2\pi G}(\nabla\phi)^2\delta_{\gamma\delta}. \tag{15}$$

So in this case $\theta^{00}$ is negative

$$\theta^{00} = -\frac{7}{8\pi G}(\nabla\phi)^2. \tag{16}$$

Turning now to conservation laws of total energy-momentum we remind first how matters stand in general relativity. There the energy-momentum tensor of point-like particles $\overset{p}{T}{}^{\mu\nu}$ is connected with its counterpart in special relativity $\mathcal{T}^{\mu\nu}$ by the relation, see (33.4), (33.5) and (106.4) in [16]:

$$\sqrt{-g}\,\overset{p}{T}{}^{\mu\nu} = \mathcal{T}^{\mu\nu} = \sum_a m_a u^\mu u^\nu \frac{ds}{dt}\delta(\vec{x} - \vec{x}_a(t)), \quad u^\mu = dx^\mu/ds, \tag{17}$$

$g$ is determinant of $g_{\mu\nu}$. In terms of $\mathcal{T}^{\mu\nu}$ the conservation laws are (see (96.1) in [16])

$$\mathcal{T}^{\mu}{}_{\nu,\mu} = [\mathcal{T}^{\mu\tau}(\eta_{\tau\nu} + h_{\tau\nu})]_{,\mu} = \frac{1}{2}h_{\mu\sigma,\nu}\mathcal{T}^{\mu\sigma}. \tag{18}$$

We shall see below that $\mathcal{T}^{\mu\tau}h_{\tau\nu}$ can be interpreted as (part of) interaction energy-momentum tensor.

As is known the equation of motion of particles in general relativity

$$\frac{d^2x^k}{ds^2} + \Gamma^k_{mj}u^m u^j = 0 \tag{19}$$

is contained in conservation laws. Indeed from

$$\overset{p}{T}{}^{jk}{}_{;j} = \overset{p}{T}{}^{jk}{}_{,j} + \Gamma^j_{mj}\overset{p}{T}{}^{mk} + \Gamma^k_{mj}\overset{p}{T}{}^{jm} = 0 \tag{20}$$

taking into account that from definition of $\overset{p}{T}{}^{jk}$ in (17)

$$\overset{p}{T}{}^{jk}{}_{,j} = -\frac{1}{2}(-g)^{-\frac{3}{2}}(-g)_{,j}\mathcal{T}^{jk} + (-g)^{-\frac{1}{2}}\mathcal{T}^{jk}{}_{,j}, \quad \Gamma^j_{mj} = \frac{1}{2g}g_{,m}, \tag{21}$$

we get

$$\mathcal{T}^{jk}{}_{,j} + \Gamma^k_{mj}\mathcal{T}^{mj} = 0. \tag{22}$$

This is equivalent to (19), because [9]

$$\mathcal{T}^{jm}{}_{,j} = \sum_a \frac{dp^m}{dt}\delta(\vec{x} - \vec{x}_a(t)). \tag{23}$$

Going back to field-theoretical approach, we rewrite the equation of motion of particles (19) in the lowest approximation

$$\frac{du^\mu}{ds} = \frac{d^2x^\mu}{ds^2} = \frac{1}{2}h_{\alpha\beta}{}^{,\mu}u^\alpha u^\beta - h^\mu{}_{\alpha,\beta}u^\alpha u^\beta. \tag{24}$$

Just at such movement of particles the divergence of total energy-momentum ought to be zero, and inversely, from zero divergence follows eq. (24). From (23) and (24) we find

$$\mathcal{T}^{\gamma\delta}{}_{,\gamma} = \frac{1}{2}h_{\alpha\beta}{}^{,\delta}\mathcal{T}^{\alpha\beta} - h^\delta{}_{\alpha,\gamma}\mathcal{T}^{\alpha\gamma} \quad , \tag{25}$$

This agrees with (22) and (18) in considered approximation. With the same accuracy this can be rewritten as

$$(\mathcal{T}^{\gamma\delta} + \mathcal{T}^{\gamma\alpha}h_\alpha{}^\delta)_{,\gamma} = \frac{1}{2}h_{\alpha\beta}{}^{,\delta}\mathcal{T}^{\alpha\beta}. \tag{26}$$

Using linearized Einstein equation for $\varphi_{\mu\nu}$

$$\begin{gathered}\varphi_{\mu\nu,\lambda}{}^\lambda - \frac{1}{2}\eta_{\mu\nu}\varphi_{,\lambda}\lambda - \varphi^\lambda{}_{\nu,\mu\lambda} - \varphi^\lambda{}_{\mu,\nu\lambda} = f\bar{\mathcal{T}}_{\mu\nu} \quad , \\ \bar{\mathcal{T}}_{\mu\nu} = \mathcal{T}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\mathcal{T} \quad , \quad \mathcal{T} = \mathcal{T}_\sigma{}^\sigma \quad ,\end{gathered} \tag{27}$$

we get

$$\overset{f}{T}{}^{\gamma\delta}{}_{,\gamma} = f\varphi^{\alpha\beta,\delta}\bar{\mathcal{T}}_{\alpha\beta} = -\frac{1}{2}\bar{h}_{\alpha\beta}{}^{,\delta}\bar{\mathcal{T}}^{\alpha\beta} = -\frac{1}{2}h_{\alpha\beta}{}^{,\delta}\mathcal{T}^{\alpha\beta}. \tag{28}$$

Addind (26) and (28) we see that the total energy-momentum tensor is conserved. Spin part of energy-momentum tensor is conserved by itself and do not contribute to conservation laws :

$$\overset{s}{T}{}^{\gamma\delta}{}_{,\gamma} = 0. \tag{29}$$

So from (26) and (28) it follows that the conserved tensor contains in itself the interaction tensor [2]

$$\overset{int}{T}{}^{\gamma\delta} = \mathcal{T}^{\gamma\alpha}h_{\alpha}{}^{\delta}. \tag{30}$$

But it is not symmetric. In order to understand the reason we have to consider the properties of $\overset{s}{T}{}^{\gamma\delta}$ in detail despite the fact that it does not take part in conservation laws written in the form of eqs.(26) and (28). According to known rules [2,18] we have

$$\overset{s}{T}{}^{jk} = -F^{jik}{}_{,i} - F^{kij}{}_{,i} - F^{ikj}{}_{,i} \quad , \tag{31}$$

$$F^{jik} = \frac{\partial L}{\partial\varphi_{\alpha\beta,j}}(\varphi^{k}{}_{\alpha}\eta^{i}{}_{\beta} - \varphi^{i}{}_{\alpha}\eta^{k}{}_{\beta}), \qquad L = \overset{f}{L}\,. \tag{32}$$

The antisymmetric part of (31) is contained only in the last term. For it we have

$$-F^{ikj}{}_{,i} = \left(\frac{\partial L}{\partial\varphi_{\alpha j,i}}\right)_{,i}\varphi^{k}{}_{\alpha} - \left(\frac{\partial L}{\partial\varphi_{\alpha k,i}}\right)_{,i}\varphi^{j}{}_{\alpha} + \frac{\partial L}{\partial\varphi_{\alpha j,i}}\varphi^{k}{}_{\alpha,i} - \frac{\partial L}{\partial\varphi_{\alpha k,i}}\varphi^{j}{}_{\alpha,i}. \tag{33}$$

The first two terms on the right hand side symmetrize the interaction tensor, the last two terms symmetrize the canonical one.

It is not seen directly from (6) and (31) that field energy-momentum tensor $\theta^{\gamma\delta}$ in (14) is symmetric. This agrees with the fact that the proof of symmetry utilizes the Euler-Lagrange equations for field which is considered as free [18]. We are interested in interacting field. So using linearized Einstein equation (27) with source, we get

$$\begin{aligned}\left(\frac{\partial L}{\partial\varphi_{\alpha j,i}}\right)_{,i}\varphi^{k}{}_{\alpha} - \left(\frac{\partial L}{\partial\varphi_{\alpha k,i}}\right)_{,i}\varphi^{j}{}_{\alpha} &= f(\bar{\mathcal{T}}^{\alpha j}\varphi^{k}{}_{\alpha} - \bar{\mathcal{T}}^{\alpha k}\varphi^{j}{}_{\alpha}) \\ &= f(\mathcal{T}^{\alpha j}\psi^{k}{}_{\alpha} - \mathcal{T}^{\alpha k}\psi^{j}{}_{\alpha}) = \frac{1}{2}(\mathcal{T}^{\alpha k}h^{j}{}_{\alpha} - \mathcal{T}^{\alpha j}h^{k}{}_{\alpha}).\end{aligned} \tag{34}$$

In the last equation we use the connection between $\psi_{\mu\nu}$ and $h_{\mu\nu}$, see (5). Substituting in (30) $\gamma \to j, \delta \to k$, we see that the sum of (30) and (34) is symmetric. This result retains if we start from another Lagrangian differing from Thirring's one in (6) by divergence because the linearized equation remains the same.

One should take into account however that the symmetric part of $\overset{s}{T}{}^{jk}$ can also contain terms of interaction type. So for the Lagrangian in (6) similarly to (34) we find

$$-F^{jik}{}_{,i}-F^{kij}{}_{,i}=[f\mathcal{T}-2\varphi_{il}{}^{,il}]\varphi^{kj}+f[\bar{\mathcal{T}}^{\alpha j}\varphi^{k}{}_{\alpha}+\bar{\mathcal{T}}^{\alpha k}\varphi^{j}{}_{\alpha}]$$
$$+2(\varphi^{ij,\alpha}{}_{i}\varphi^{k}{}_{\alpha}+\varphi^{ki,\alpha}{}_{i}\varphi^{j}{}_{\alpha})+2\varphi^{j\alpha,i}\varphi^{k}{}_{\alpha,i}-(\varphi^{\alpha i,j}\varphi^{k}{}_{\alpha,i}+\varphi^{\alpha i,k}\varphi^{j}{}_{\alpha,i})$$
$$-2(\varphi^{jk,\alpha}{}_{i}\varphi^{i}{}_{\alpha}+\varphi^{jk,\alpha}\varphi^{i}{}_{\alpha,i})+2\varphi^{ki,\alpha}\varphi^{j}{}_{\alpha,i}. \quad (35)$$

As a result we get for $\overset{s}{T}{}^{jk}$

$$\overset{s}{T}{}^{jk}=2[(\varphi^{ij,\alpha}{}_{i}\varphi^{k}{}_{\alpha}+\varphi^{ik,\alpha}{}_{i}\varphi^{j}{}_{\alpha})-\varphi^{i\alpha}{}_{,i\alpha}\varphi^{kj}+\varphi^{j\alpha,i}\varphi^{k}{}_{\alpha,i}-\varphi^{jk,\alpha}{}_{i}\varphi^{i}{}_{\alpha}$$
$$-\varphi^{jk,\alpha}\varphi^{i}{}_{\alpha,i}+\varphi^{ki,\alpha}\varphi^{j}{}_{\alpha,i}]-2\varphi^{i\alpha,j}\varphi^{k}{}_{\alpha,i}+2f\mathcal{T}^{j\alpha}\varphi^{k}{}_{\alpha}. \quad (36)$$

Here last but one term, added to $\overset{f}{T}{}^{jk}$, makes it symmetric. The last term can be rewritten in terms of $h_{\mu\nu}$ in the form, see (9) and (11), (30),

$$-\mathcal{T}^{j\alpha}(h_{\alpha}{}^{k}-\frac{1}{2}\eta_{\alpha}{}^{k}h)=-\overset{int}{T}{}^{jk}+\frac{1}{2}\mathcal{T}^{jk}h. \quad (37)$$

So the symmetrization of $\overset{int}{T}{}^{jk}$ in (30) is reduced to its replacement by $\frac{1}{2}\mathcal{T}^{jk}h$. This tensor is nonzero only where particles are present. For Newtonian centers the corresponding energy density

$$\frac{1}{2}\mathcal{T}^{00}h=-2\mathcal{T}^{00}\phi \quad (38)$$

is positive (contrary to our intuition and) contrary to $\overset{int}{T}{}^{00}$ in (30), see (12) and (10), where $h$ and $\phi$ are given for Newtonian centers.

We note that the use of linearized Einstein eq. (27) in the expression for $\overset{s}{T}{}^{jk}$ leads to that eq. (29) is satisfied only with considered accuracy. The presence of interaction energy-momentum tensor means the appearance of such vertex: the energy-momentum tensor of matter together with one of gravitons serves as a source for other graviton, see Fig.1

Now we note that Belinfante-Rosenfeld procedure leads to the appearance in gravitational energy-momentum tensor terms with second derivatives.

Thirring assumes that his tensor $\theta^{\gamma\delta}$ (see (14), (6), (31)) is an analog of energy-momentum tensor figuring in the r.h.s. of Einstein equation when iteration procedure is used. In other words the nonlinear correction to field is given only by diagram (2a) in Fig.2. On this Fig. the short straight line has only conditional meaning: it represents the source of gravitons, namely the energy-momentum tensor build of two gravitons (real or virtual) shown on Fig.2 as joining the ends of this line. The graviton emerging from the middle of straight line is emitted or absorbed by this source. On diagram (2a) the energy-momentum tensor is build from gravitons of Newtonian center. On diagrams

(2b) and (2c) one of the virtual gravitons of Newtonian center interact with energy-momentum tensor of two other gravitons. All three diagrams on Fig.2 correspond to one Feynman diagram obtained by contracting the short straight line to a point.

The contribution to nonlinear correction for field from diagram (2a) is easy to obtain. Indeed, from (14), (6) and (15) we have

$$\theta^{jk} = \overset{f}{T}{}^{jk} = \frac{1}{4\pi G}\left(\phi_{,j}\phi_{,k} - -\frac{\delta_{jk}}{2}(\nabla\phi)^2\right), \quad j,k = 1,2,3. \tag{39}$$

Using now the field equation in Hilbert gauge with $\theta^{\mu\nu}$ from (16) and (39)

$$\Box\bar{h}^{\mu\nu} = -16\pi G\theta^{\mu\nu}, \quad \Box = \frac{\partial^2}{\partial t^2} - \nabla^2 \tag{40}$$

we find

$$\bar{h}^{00} = -7\phi^2, \quad \bar{h}^{ik} = -\frac{G^2M^2}{r^4}x^i x^k, \quad \phi = -\frac{GM}{r}, \quad i,k = 1,2,3. \tag{41}$$

Here easily verifiable relations

$$\nabla^2\frac{x^i x^k}{r^4} = \frac{2\delta_{ik}}{r^4} - \frac{4x^i x^k}{r^6}, \quad \nabla^2\frac{1}{r^2} = \frac{2}{r^4} \tag{42}$$

were used. The obtained $\bar{h}^{\mu\nu}$ satisfies Hilbert condition. Going over to $h_{\mu\nu} = \bar{h}^{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}$, we find the following nonlinear corrections

$$h_{00} = -4\phi^2, \quad h_{ik} = -G^2M^2(\frac{x_i x_k}{r^4} + \frac{3\delta_{ik}}{r^2}). \tag{43}$$

Index 2 in $\overset{(2)}{h}{}_{\mu\nu}$, indicating the order of correction in powers of $G$, is dropped for brevity.

Finally from (12) and (43) we have

$$ds^2 = g_{00}dt^2 - (1 - 2\phi + 3\phi^2)\delta_{ik}dx^i dx^k - \phi^2\frac{x_i x_k dx^i dx^k}{r^2}, \tag{44}$$

$$g_{00} = 1 + 2\phi - 4\phi^2. \tag{45}$$

The transition to spherical coordinates is given by the relations

$$\delta_{ik}dx^i dx^k = dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad \frac{x_i x_k dx^i dx^k}{r^2} = (dr)^2.$$

Why the nonlinear correction in (45) turns out to be negative? It will appear later on that it is caused solely by the source (15), see eqs. (95) and (93a). The sources (8) and (15) have different signs, but the corresponding fields have the

same sign. The answer is simple. The correction in (45) is only a small (and negative) part. The larger and positive part goes for converting initially bare mass in Newtonian potential into a dressed one, see eq. (A9). Now the negative sign of correction is clear: the mass of Newtonian center at infinitely large distance appears as $M$, but at finite distance the test particle feels a greater mass and greater attraction, because (15) is negative.

The nonlinear correction $-4\phi^2$ in $g_{00}$ in (45) is of special interest for us. The correct value necessary to explain the perihelion shift is $+2\phi^2$. The shortest way to see this is to use the method described in §101 in [16]. We write in spherical coordinates

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2).$$

The solution to Hamilton-Jacobi equation has the form

$$S = -\mathcal{E}t + J\varphi + S_r(r), \quad S_r(r) = \int B(r)[\frac{\mathcal{E}^2}{A(r)} - \frac{J^2}{C(r)} - m^2]^{1/2} dr.$$

Here $\mathcal{E}$ and $J$ are constants.

For nonrelativistic particle $\mathcal{E} = m + \mathcal{E}'$, $\mathcal{E}' \ll m$, and the main terms in square bracket in the expression for $S_r$ are cancelled out:

$$\frac{1}{A(r)} - 1 \approx -2\phi(1-\phi),$$

where we have assumed that $A(r) = 1 + 2\phi + 2\phi^2$.

So we have to retain in $A^{-1}(r)$ terms of order $\phi^2$, but in $B(r)$ and $r^2C(r)^{-1}$ only terms of order $\phi$: $B(r) = r^{-2}C(r) = 1 - 2\phi$. Thus

$$B(r)(A(r)^{-1} - 1) \approx -(1-2\phi)2\phi(1-\phi) \approx -2\phi(1-3\phi) = \frac{r_g}{r} + \frac{3r_g^2}{2r^2}.$$

The leading term $\propto r^{-2}$ in $S_r$ is

$$-\frac{1}{r^2}(J^2 - \frac{3m^2r_g^2}{2}).$$

As explained in [16], the expansion

$$S_r(J^2 - \frac{3m^2r_g^2}{2}) = S_r(J^2) - \frac{3m^2r_g^2}{2}\frac{\partial S_r}{\partial J^2}$$

directly leads to correct perihelion shift.

In general the motion of a particle is described by equations (cf. §4 in Ch.8 in [9])

$$C(r)\frac{d\varphi}{dt} = JA(r), \quad \frac{B(r)}{A^2(r)}\left(\frac{dr}{dt}\right)^2 + \frac{J^2}{C(r)} - \frac{1}{A(r)} = -E. \qquad (45a)$$

For nonrelativistic particle only the third term on the l.h.s. of second equation in (45a) does not contain small factor of order $v^2$. So it requires more accurate approximation. For relativistic particle $A(r)$, $B(r)$ and $\frac{C(r)}{r^2}$ ought to be considered in the same approximation.

Now taking into account all 3 diagrams of Fig.2 we get instead of (44) (see Appendix for more details and pay attention to difference in metric signature there)

$$ds^2 = (1 + 2\phi + 4\phi^2)dt^2 - (1 - 2\phi + 9\phi^2)(d\vec{x})^2 - 13\phi^2 \frac{x^i x^j}{r^2} dx^i dx^j, \qquad (45b)$$

As the nonlinear correction to $g_{00}$ is twice as much as necessary, Thirring tensor alone is insufficient.

We note here that Thirring obtained from his tensor the necessary correction. Yet his result is objectionable as he used illdefined gauge

$$\Box^2 \Lambda = \frac{G^2 M^2}{4r^2},$$

see eq. (83) in [2]. Namely the source of $\Lambda$ fall of too slowly for large $r$ and the integral defining $\Lambda$ , see (A8), diverges for large $r'$.

In the next two Sections we shall see how energy-momentum "tensors" of general relativity differ from Thirring's tensor.

# 3 Landau-Lifshitz pseudotensor of energy-momentum

This pseudotensor in the sense and approximation considered here is a tensor. In the lowest approximation with help of relation

$$\sqrt{-g} g^{ik} \approx (1 + \frac{h}{2})(\eta^{ik} - h^{ik}) \approx \eta^{ik} - \bar{h}^{ik}. \qquad (46)$$

we get from eq. (96.9) in [16]

$$t^{ik} = \frac{1}{16\pi G}[\bar{h}^{ik,l}\bar{h}_{lm}{}^{,m} - \bar{h}^{il}{}_{,l}\bar{h}^{km}{}_{,m} - \bar{h}^{km,p}\bar{h}_{mp}{}^{,i} - \bar{h}^{im,p}\bar{h}_{mp}{}^{,k} + \bar{h}^{im,p}\bar{h}^{k}{}_{m,p} + \frac{1}{2}\bar{h}^{pq,i}\bar{h}_{pq}{}^{,k} - \frac{1}{4}\bar{h}^{,i}\bar{h}^{,k} + \eta^{ik}(\frac{1}{2}\bar{h}_{mn,p}\bar{h}^{pm,n} + \frac{1}{8}\bar{h}^{,m}\bar{h}_{,m} - \frac{1}{4}\bar{h}_{pq,m}\bar{h}^{pq,m})]. \qquad (47)$$

Comparison with canonical tensor (6) shows that it is connected with $t^{ik}$ by the relation

$$t^{ik} = \overset{f}{T}{}^{ik} + F^{ik}, \quad \bar{h}_{ik} = -2f\varphi_{ik},$$

$$F^{ik} = \frac{1}{16\pi G}[\bar{h}^{ik,l}\bar{h}_{ln}{}^{,n} - \bar{h}^{il}{}_{,l}\bar{h}^{kn}{}_{,n} - \bar{h}^{kn,p}\bar{h}_{np}{}^{,i} + \bar{h}^{in,p}\bar{h}^{k}{}_{n,p}]. \qquad (48)$$

From (14) we see that now in place of $\overset{s}{T}{}^{ik}$ stands $F^{ik}$. But $\overset{s}{T}{}^{ik}$ was a conserved quantity, see.(29). So $F^{ik}$ should rather play the role of interaction energy-momentum tensor. Indeed, taking into account that in considered approximation $\bar{h}_{ik}$ satisfies the linearized Einstein equation

$$\bar{h}_{np,j}{}^{j} - \bar{h}_{jp,n}{}^{j} - \bar{h}_{jn,p}{}^{j} + \eta_{np}\bar{h}_{qr}{}^{,qr} = -16\pi G\mathcal{T}_{np}, \tag{49}$$

we find

$$F^{jk}{}_{,j} = \bar{h}^{kn,i}\mathcal{T}_{ni} = h^{kn,i}\mathcal{T}_{ni} - \frac{1}{2}h^{,i}\mathcal{T}_{i}{}^{k}. \tag{50}$$

Now we check that coservation laws [16]

$$\frac{\partial}{\partial x^k}\left((-g)[\overset{p}{T}{}^{ik} + t^{ik}]\right) = 0 \tag{51}$$

are fulfilled. From (48), (28) and (50) we have

$$t^{ik}{}_{,i} = -\frac{1}{2}h^{iq,k}\mathcal{T}_{iq} + h^{kn,i}\mathcal{T}_{ni} - \frac{1}{2}h^{,i}\mathcal{T}_{i}{}^{k}. \tag{52}$$

For matter energy-momentum tensor from (17) we get

$$(-g)\,\overset{p}{T}{}^{ik} = \sqrt{-g}\mathcal{T}^{ik} \approx (1 + \frac{h}{2})\mathcal{T}^{ik}, \quad -g \approx 1 + h. \tag{53}$$

From here

$$\left((-g)\,\overset{p}{T}{}^{ik}\right)_{,i} = \mathcal{T}^{ik}{}_{,i} + \frac{1}{2}h_{,i}\mathcal{T}^{ik}, \tag{54}$$

the terms of order $h^2$ being dropped. Now it follows from (25), (52) and (54) that (51) is fulfilled. As seen from (53) here too there is a tensor, which is nonzero only where particles are located. Surprisingly it coincides with Thirring's interaction tensor, see (37) and text below it.

Although $-g\,\overset{p}{T}{}^{ik} + t^{ik}$ differs from Thirring's $\mathcal{T}^{ik} + \overset{int}{T}{}^{ik} + \theta^{ik}$, for Newtonian centers they coincide, see eqs. (16) and (39).

Now we turn to Newtonian approximation. According to Problem 1 in §106 in [16] the energy density of gravitational field in this approximation is given by $\overset{f}{T}{}^{00}$ in (13), (10). But there is also energy density of interaction $\mu\phi$, where $\mu$ is density of particles. Using Poisson equation $\nabla^2\phi = 4\pi G\mu$ and ignoring the problems connected with point-like nature of particles, we can write (utilizing integration by parts)

$$\int \mu\phi dV = -\int \frac{1}{4\pi G}(\nabla\phi)^2 dV, \tag{55}$$

The density in the integrand on the r.h.s. contains now not only the energy density of interaction, but also the proper energy density of particle's self-field.

The density on the l.h.s. is nonzero only at particle locations, the density on the r.h.s. is nonzero where the field is nonzero. The integration by parts deprive us the possibility to retain the previous physical meaning of integrand. If nevertheless we do this, then adding to (13) the energy density in the r.h.s. of (55) we get the effective gravitational energy density in Newtonian approximation

$$-\frac{1}{8\pi G}(\nabla\phi)^2. \tag{56}$$

To bring this in agreement with $t^{00}$ we ought, according to a foot-note in [16], take into account the contribution from $(-g)\overset{p}{T}{}^{00}$. Let us do it. For $t^{00}$ we have

$$t^{00} = -\frac{7}{8\pi G}(\nabla\phi)^2, \tag{57}$$

where $\phi$ is the potential of Newtonian centers. Now

$$(-g)\overset{p}{T}{}^{00} = \sqrt{-g}\mathcal{T}^{00} \approx \mathcal{T}^{00}(1+\frac{h}{2}) \approx \mathcal{T}^{00} - 2\phi\mu, \tag{58}$$

see (12). The sought for agreement will be reached only after we rewrite a la Thirring [2] $\mathcal{T}^{00}$ in terms of observables. From (17)

$$\mathcal{T}^{00} = \sum_a m_a \frac{dx^0}{ds}\delta(\vec{x}-\vec{x}_a(t)). \tag{59}$$

In the presence of gravitational field

$$ds^2 = g_{00}dt^2(1-v^2). \tag{60}$$

Here $v^2$ is physical velocity, see §88 in [16]. Hence

$$\frac{dx^0}{ds} = \frac{1}{\sqrt{g_{00}(1-v^2)}} \approx \frac{1}{\sqrt{1-v^2}} - \frac{h_{00}}{2}, \quad v^2 \ll 1. \tag{61}$$

Thus after going over to the observable velocity we obtain the term

$$-\frac{h_{00}}{2}\mu = -\phi\mu. \tag{62}$$

detached from $\mathcal{T}^{00}$. Equation (12) was used to get the r.h.s.. Together with corresponding term in (58) this leads to

$$-3\mu\phi \Longrightarrow \frac{3}{4\pi G}(\nabla\phi)^2, \tag{63}$$

where arrow corresponds to going over in (55) from integrand on the l.h.s. to the integrand on r.h.s.. Now the sum of (57) and (63) gives the expected (56).

The consideration of Newtonian approximation makes the following point of view very enticing: The energy density of an isolated point-like particle should be positive; Hilbert gauge exclude the unnecessary spins and then positivity seems quite natural, because the presence of virtual gravitons should not make the energy density negative. The attraction is described by interaction energy density and so the latter must be negative. Neither Thirring tensor nor Landau-Lifshitz one satisfies this requirement. The MTW tensor does. Unfortunately I failed to fit this idea into existing approach to gravitation.

Using LL tesor in 3-graviton vertex we get from diagram (2a) the same contribution as in the case of Thirring tensor. The contribution from all 3 diagrams of Fig. 2 leads to

$$ds^2 = (1+2\phi+4\phi^2)dt^2 - (1-2\phi+7\phi^2)(d\vec{x})^2 + 7\phi^2\frac{x^i x^j}{r^2}dx^i dx^j. \qquad (63a)$$

# 4 Papapetrou- Weinberg energy-momentum tensor.

Einstein equation can be recast in such a way that gravitational energy -momentum "tensor" can be easily identified in coordinate system that goes over to Minkowski system at large distances from gravitating bodies [9]. In the lowest approximation this tensor have the form, see eq. (7.6.14) in [9])

$$t_{\mu\kappa} = \frac{1}{8\pi G}[-\frac{1}{2}h_{\mu\kappa}R^{(1)} + \frac{1}{2}\eta_{\mu\kappa}h^{\rho\sigma}R^{(1)}_{\rho\sigma} + R^{(2)}_{\mu\kappa} - \frac{1}{2}\eta_{\mu\kappa}R^{(2)}],\ R^{(1,2)} = R^{(1,2)\mu}_{\mu}. \qquad (64)$$

In this Section we use Weinberg notation:

$$\eta_{\mu\nu} = diag(-1,1,1,1),\ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},\ h_{\mu\nu} \equiv h^W_{\mu\nu} = -h^{Thir} = -h^{LL}_{\mu\nu}. \qquad (65)$$

Greek indices run from 0 to 3, latin ones from 1 to 3. $R^{(1,2)}_{\mu\nu}$ is Ricci tensor in the first and second approximation in powers of $h_{\mu\nu}$. The indices are raised and lowered with $\eta_{\mu\nu}$. In terms of $\bar{h}_{\mu\nu}$ we get

$$R^{(1)}_{\mu\nu} = \frac{1}{2}(\bar{h}_{\mu\nu,\sigma}{}^{\sigma} - \bar{h}_{\sigma\mu,\nu}{}^{\sigma} - \bar{h}_{\sigma\nu,\mu}{}^{\sigma}) - \frac{1}{4}\eta_{\mu\nu}\bar{h}_{,\sigma}{}^{\sigma}\ ,\ R^{(1)} = -\bar{h}^{\mu\sigma}{}_{,\mu\sigma} - \frac{1}{2}\bar{h}_{,\sigma}{}^{\sigma}\ , \qquad (66)$$

$$\begin{aligned}R^{(2)}_{\mu\kappa} = {} & \frac{1}{2}\bar{h}^{\lambda\nu}[\bar{h}_{\mu\nu,\kappa\lambda} + \bar{h}_{\kappa\nu,\mu\lambda} - \bar{h}_{\lambda\nu,\kappa\mu} - \bar{h}_{\mu\kappa,\nu\lambda}] - \frac{1}{4}(\bar{h}^{\lambda}{}_{\mu}\bar{h}_{,\kappa\lambda} + \bar{h}_{\kappa}{}^{\nu}\bar{h}_{,\mu\nu}) \\ & + \frac{1}{4}\bar{h}(\bar{h}_{,\mu\kappa} + \bar{h}_{\mu\kappa,\lambda}{}^{\lambda} - \bar{h}_{\mu}{}^{\lambda}{}_{,\kappa\lambda} - \bar{h}^{\nu}{}_{\kappa,\nu\mu}) - \frac{1}{4}(\bar{h}^{\nu}{}_{\mu,\nu}\bar{h}_{,\kappa} + \bar{h}^{\nu}_{\kappa,\nu}\bar{h}_{,\mu}) + \\ & \bar{h}^{,\sigma}(\frac{1}{2}\bar{h}_{\mu\kappa,\sigma} - \frac{1}{4}\bar{h}_{\kappa\sigma,\mu} - \frac{1}{4}\bar{h}_{\mu\sigma,\kappa}) + \frac{1}{2}\bar{h}^{\nu}{}_{\sigma,\nu}(\bar{h}^{\sigma}{}_{\mu,\kappa} + \bar{h}^{\sigma}{}_{\kappa,\mu} \\ & -\bar{h}_{\mu\kappa}{}^{,\sigma}) + \frac{1}{2}\bar{h}_{\kappa\sigma,\lambda}(\bar{h}_{\mu}{}^{\lambda,\sigma} - \bar{h}_{\mu}{}^{\sigma,\lambda}) - \frac{1}{4}\bar{h}_{\sigma\lambda,\kappa}\bar{h}^{\sigma\lambda}{}_{,\mu} + \frac{1}{8}\bar{h}_{,\mu}\bar{h}_{,\kappa}\end{aligned}$$

$$+\eta_{\mu\kappa}[\frac{1}{4}\bar{h}^{\lambda\nu}\bar{h}_{,\lambda\nu} - \frac{1}{8}\bar{h}\bar{h}_{,\lambda}{}^{\lambda} + \frac{1}{4}\bar{h}^{\nu}{}_{\sigma,\nu}\bar{h}^{,\sigma} - \frac{1}{8}\bar{h}_{,\sigma}\bar{h}^{,\sigma}], \quad (67)$$

$$R^{(2)} = \bar{h}^{\lambda\nu}(\bar{h}_{\kappa\nu}{}^{,\kappa}{}_{\lambda} - \frac{1}{2}\bar{h}_{\nu\lambda,\sigma}{}^{\sigma}) + \bar{h}^{\nu}{}_{\sigma,\nu}\bar{h}^{\sigma\kappa}{}_{,\kappa} - \frac{1}{2}\bar{h}^{\nu}{}_{\sigma,\nu}\bar{h}^{,\sigma}$$
$$+\frac{1}{2}\bar{h}_{\kappa\sigma,\lambda}\bar{h}^{\kappa\lambda,\sigma} - \frac{3}{4}\bar{h}_{\kappa\sigma,\lambda}\bar{h}^{\kappa\sigma,\lambda} - \frac{1}{2}\bar{h}\bar{h}^{\kappa\lambda}{}_{,\kappa\lambda} + \frac{1}{8}\bar{h}_{,\sigma}\bar{h}^{,\sigma}. \quad (68)$$

For Newtonian center from (9-12) we obtain

$$\bar{h}_{\mu\nu} = -\bar{h}^{T}_{\mu\nu} = -4\phi\delta_{\mu0}\delta_{\nu0},\ h_{\mu\nu} = -h^{T}_{\mu\nu} = -2\phi\delta_{\mu\nu},\ h = h^{W} = h^{T} = -4\phi = -\bar{h}. \quad (69)$$

Nonzero components of $t_{\mu\kappa}$ are

$$t_{00} = -\frac{3}{8\pi G}(\nabla\phi)^2 = -\frac{3GM^2}{8\pi r^4},\ t_{ik} = \frac{GM^2}{8\pi r^6}(7\delta_{ik}r^2 - 14x^i x^k). \quad (70)$$

In Hilbert gauge from equation

$$\nabla^2\bar{h}_{\mu\nu} = -16\pi G t_{\mu\nu} \quad (71)$$

we find, cf. with (40), (42),

$$\bar{h}_{00} = \frac{3G^2M^2}{r^2} = 3\phi^2,\ \bar{h}_{ik} = -7G^2M^2\frac{x^i x^k}{r^4}. \quad (72)$$

It is easy to check that (72) satisfies the Hilbert condition (2). In terms of $h_{\mu\nu}$ we have

$$h_{00} = -2\phi^2,\ h_{ik} = G^2M^2(\frac{5\delta_{ik}}{r^2} - 7\frac{x^i x^k}{r^4}). \quad (73)$$

In the expressions (71-73) $h_{\mu\nu}$ is nonlinear correction.

On the other hand in harmonic coordinates the Schwarzschild solution has the form [9]

$$-d\tau^2 = -\frac{1+\phi}{1-\phi}dt^2 + (1-\phi)^2(d\vec{x})^2 + \frac{1-\phi}{1+\phi}\phi^2\frac{x^i x^k}{r^2}dx^i dx^k. \quad (74)$$

So in the considered approximation this gives

$$g_{00} = -(1 + 2\phi + 2\phi^2), \quad (75)$$

$$g_{ik} = (1-2\phi)\delta_{ik} + \phi^2(\delta_{ik} + \frac{x_i x_k}{r^2}). \quad (76)$$

From (69) we have $h^{(1)}_{00} = -2\phi$, from (73) $h^{(2)}_{00} = -2\phi^2$, and there is agreement with (75). As to the nonlinear correction for $g_{ik}$, in (73) it differs from the one

in (76) by a gauge. Really, subtracting from $h_{ik}$ in (73) the nonlinear part of (76), we find

$$G^2M^2(\frac{4\delta_{ik}}{r^2} - \frac{8x_ix_k}{r^4}) = 2G^2M^2(\Lambda_{i,k} + \Lambda_{k,i}),\ \Lambda_i = \frac{x_i}{r^2}, \tag{77}$$

i.e. a gauge.

Going back to $t_{00}$ in (70), we note that this density is negative and does not coincide with any density of other tensors. At the same time the equation of motion of particles is contained in the conservation laws of total energy-momentum tensor. We shall check it in considered approximation. For gravitational part the calculations give

$$t^{\mu\kappa}{}_{,\kappa} = -h^{\nu}{}_{\sigma,\nu}\mathcal{T}^{\mu\sigma} + \frac{1}{2}h_{,\sigma}\mathcal{T}^{\mu\sigma} - \frac{1}{2}h^{\rho\sigma,\mu}\mathcal{T}_{\rho\sigma} - h^{\nu\lambda}\mathcal{T}^{\mu}{}_{\nu,\lambda}. \tag{78}$$

The energy-momentum tensor for particles, figuring in conservation laws, has a rather complicated form by construction [9]

$$\begin{aligned}\tau^{\mu\kappa} = \eta^{\mu\sigma}\eta^{\kappa\tau}g_{\sigma\alpha}g_{\tau\beta}\overset{p}{T}{}^{\alpha\beta} &\approx \overset{p}{T}{}^{\mu\kappa} + h^{\mu}{}_{\alpha}\mathcal{T}^{\alpha\kappa} + h^{\kappa}{}_{\alpha}\mathcal{T}^{\alpha\mu} \\ &\approx \mathcal{T}^{\mu\kappa} - \frac{1}{2}h\mathcal{T}^{\mu\kappa} + h^{\mu}{}_{\alpha}\mathcal{T}^{\alpha\kappa} + h^{\kappa}{}_{\alpha}\mathcal{T}^{\alpha\mu}.\end{aligned} \tag{79}$$

From here with the considered accuracy

$$\tau^{\mu\kappa}{}_{,\kappa} = \overset{p}{T}{}^{\mu\kappa}{}_{,\kappa} + h^{\mu}{}_{\alpha,\kappa}\mathcal{T}^{\alpha\kappa} + h^{\kappa}{}_{\alpha,\kappa}\mathcal{T}^{\alpha\mu} + h^{\kappa}{}_{\alpha}\mathcal{T}^{\alpha\mu}{}_{,\kappa}. \tag{80}$$

So

$$(\tau^{\mu\kappa} + t^{\mu\kappa})_{,\kappa} = \overset{p}{T}{}^{\mu\kappa}{}_{,\kappa} - \frac{1}{2}h^{\rho\sigma,\mu}\mathcal{T}_{\rho\sigma} + h^{\mu\alpha,\kappa}\mathcal{T}_{\alpha\kappa} + \frac{1}{2}h_{,\sigma}\mathcal{T}^{\mu\sigma}. \tag{81}$$

Further from (53) we get

$$\overset{p}{T}{}^{\mu\kappa}{}_{,\kappa} \approx \mathcal{T}^{\mu\kappa}{}_{,\kappa} - \frac{1}{2}h_{,\kappa}\mathcal{T}^{\mu\kappa}. \tag{82}$$

Taking into account (25) we see that the r.h.s. of (81) is zero.

Now we turn to Newtonian approximation. Terms of interaction tensor are contained in both $\tau^{\mu\nu}$ (three last terms in the r.h.s. of (79)) and in $t_{\mu\kappa}$. From (64), (66-68), using (49), which preserve its form in the notation of this Section, we find the following terms of interaction tensor contained in $t_{\mu\kappa}$ :

$$-\frac{1}{2}h_{\mu\kappa}\mathcal{T} - \eta_{\mu\kappa}(\bar{h}^{\rho\sigma}\mathcal{T}_{\rho\sigma} - \frac{1}{4}\bar{h}\mathcal{T}) - \frac{1}{2}\bar{h}\mathcal{T}_{\mu\kappa}.$$

From here and the first equation in (70) we get in Newtonian approximation

$$t_{00} = -\frac{3}{8\pi G}(\nabla\phi)^2 - 6\phi\mathcal{T}_{00}. \tag{83}$$

Here $\phi$ is the same as in (10). From (79) and (53) we get in this approximation

$$\tau^{00} = (-g)^{-\frac{1}{2}}\mathcal{T}^{00} + 2h^0{}_0\mathcal{T}^{00} \approx \mathcal{T}^{00}(1 - \frac{1}{2}h) - 2h_{00}\mathcal{T}^{00} = \mathcal{T}^{00} + 6\phi\mathcal{T}^{00}. \quad (84)$$

Thus in Newtonian approximation the interaction terms in the sum of (83) and (84) are cancelled out. The agreement with Newtonian approximation (56) is achieved in the same way as for Landau-Lifshitz tensor: $\mathcal{T}^{00}$ on the r.h.s. of (84) detaches term (62), which is equivalent (in accordance with (63)) to $\frac{1}{4\pi G}(\nabla\phi)^2$. Together with the first term on the r.h.s. of (83) this gives (56).

Weinberg shows in detail that his energy-momentum tensor has all required characteristics. But this tensor does not help us to find energy-momentum tensor of two gravitons as represented by straight line on diagrams of Fig.2. By construction Weinberg‘s tensor gives the gravitational field only via diagram (2a). The field-theoretical description tell us that test particle is not quite passive. It does not simply follows the command "move along geodesic" but itself takes part in the creation of field in which it moves, see Fig.(2b, 2c). From this viewpoint one can expect that e.g. photon and graviton scatter differently on Newtonian center as only in the latter case all three diagrams of Fig.2 contribute.

# 5 Misner-Thorne-Wheeler energy-momentum tensor

In this Section it is handy for us to use again Thirring‘s notation. Up to divergence terms the Lagrangian (6) may be rewritten as [6]

$$\mathcal{L} = \frac{1}{2}\varphi_{\mu\nu,\lambda}\varphi^{\mu\nu,\lambda} - \frac{1}{4}\varphi_{,\lambda}\varphi^{,\lambda} - \varphi_{\mu\nu}{}^{,\mu}\varphi^{\lambda\nu}{}_{,\lambda}. \quad (85)$$

The corresponding canonical energy-momentum tensor

$$\overset{f}{T}{}^{jk} = \frac{\partial\mathcal{L}}{\partial\varphi^{\mu\nu}{}_{,j}}\varphi^{\mu\nu,k} - \eta^{jk}\mathcal{L}, \quad (86)$$

acquires the form

$$\overset{f}{T}{}^{jk} = \bar{T}^{jk} - \frac{1}{2}\eta^{jk}\bar{T}\ , \bar{T}^{jk} = \varphi^{\mu\nu,k}\varphi_{\mu\nu}{}^{,j} - \frac{1}{2}\varphi^{,k}\varphi^{,j} - 2\varphi^{j\nu,k}\varphi_{\nu\sigma}{}^{,\sigma}\ , \bar{T} = -T = 2\mathcal{L}. \quad (87)$$

Spin part is given by (31-32) with substitution $L \to \mathcal{L}$. We dwell on differences from Thirring‘s tensor. In symmetric in $jk$ tensor

$$F^{jik} + F^{kij} = (\varphi^{\alpha i,j} - \varphi^{\alpha\sigma}{}_{,\sigma}\eta^{ji})\varphi^k{}_\alpha + (\varphi^{\alpha i,k} - \varphi^{\alpha\sigma}{}_{,\sigma}\eta^{ki})\varphi^j{}_\alpha$$
$$-2\varphi^{i\sigma}{}_{,\sigma}\varphi^{kj} + (2\varphi^{\alpha\sigma}{}_{,\sigma}\eta^{jk} - \varphi^{\alpha k,j} - \varphi^{\alpha j,k})\varphi^i{}_\alpha + \varphi^{k\sigma}{}_{,\sigma}\varphi^{ij} + \varphi^{j\sigma}{}_{,\sigma}\varphi^{ik} \quad (88)$$

there is no derivatives over $x^i$. This means that in divergence $F^{jik}{}_{,i} + F^{kij}{}_{,i}$ there are no terms of interaction tensor. In antisymmetric in $jk$ tensor

$$F^{ikj} = (\varphi^{\alpha k,i} - \varphi^{\alpha\sigma}{}_{,\sigma}\eta^{ik})\varphi^{j}{}_{\alpha} - \varphi^{k\sigma}{}_{,\sigma}\varphi^{ij} + (\varphi^{\alpha\sigma}{}_{,\sigma}\eta^{ij} - \varphi^{\alpha j,i})\varphi^{k}{}_{\alpha} + \varphi^{j\sigma}{}_{,\sigma}\varphi^{ik} \quad (89)$$

such terms are present. Hence, using linearized Einstein equation (27) in the expression for $F^{ikj}{}_{,i}$, we get

$$-F^{ikj}{}_{,i} = f(\bar{\mathcal{T}}^{j\alpha}\varphi^{k}{}_{\alpha} - \bar{\mathcal{T}}^{k\alpha}\varphi^{j}{}_{\alpha}) + \varphi^{\alpha\sigma}{}_{,\sigma}(\varphi^{j}{}_{\alpha}{}^{,k} - \varphi^{k}{}_{\alpha}{}^{,j}). \quad (90)$$

Terms with $f$ together with (30) give symmetric interaction tensor in accordance with (34). Other two terms on the r.h.s. of (90) supplement $\overset{f}{T}{}^{jk}$ to symmetric one, see (87). So we get

$$\begin{aligned}\theta^{jk} = \overset{f}{T}{}^{jk} + \overset{s}{T}{}^{jk} = \varphi^{\mu\nu,k}\varphi_{\mu\nu}{}^{,j} - \frac{1}{2}\varphi^{,k}\varphi^{,j} - \varphi^{j\sigma}{}_{,\sigma i}\varphi^{ik} - \varphi^{k\sigma}{}_{,\sigma i}\varphi^{ij} \\ -\varphi^{\alpha i,j}\varphi^{k}{}_{\alpha,i} - \varphi^{\alpha i,k}\varphi^{j}{}_{\alpha,i} + (\varphi^{\alpha j,k}{}_{i} + \varphi^{\alpha k,j}{}_{i})\varphi^{i}{}_{\alpha} + 2\varphi^{i\sigma}{}_{,i\sigma}\varphi^{jk} + \\ 2\varphi^{i\sigma}{}_{,\sigma}\varphi^{kj}{}_{,i} - 2\varphi^{j\sigma}{}_{,\sigma}\varphi^{ki}{}_{,i} + (\varphi^{\alpha k,j} + \varphi^{\alpha j,k})\varphi_{\alpha\sigma}{}^{,\sigma} \\ -2\eta^{jk}(\varphi^{\alpha\sigma}{}_{,\sigma}\varphi^{i}{}_{\alpha})_{,i} - \eta^{jk}\mathcal{L} + f(\bar{\mathcal{T}}^{j\alpha}\varphi^{k}{}_{\alpha} - \bar{\mathcal{T}}^{k\alpha}\varphi^{j}{}_{\alpha}).\end{aligned} \quad (91)$$

From (91) and (30), using the relation between $\varphi_{\mu\nu}$ and $\bar{h}_{\mu\nu}$ in (9), and taking into account (34), we find

$$\begin{aligned}\theta^{jk} + \overset{int}{T}{}^{jk} = \frac{1}{32\pi G}[\bar{h}^{\mu\nu,k}\bar{h}_{\mu\nu}{}^{,j} - \frac{1}{2}\bar{h}^{,k}\bar{h}^{,j} - (\bar{h}^{j\sigma}{}_{,\sigma i}\bar{h}^{ik} + \bar{h}^{k\sigma}{}_{,\sigma i}\bar{h}^{ij}) \\ -(\bar{h}^{\alpha i,j}\bar{h}^{k}{}_{\alpha,i} + \bar{h}^{\alpha i,k}\bar{h}^{j}{}_{\alpha,i}) + (\bar{h}^{\alpha j,k}{}_{i} + \bar{h}^{\alpha k,j}{}_{i})\bar{h}^{i}{}_{\alpha} + 2\bar{h}^{i\sigma}{}_{,\sigma i}\bar{h}^{jk} + 2\bar{h}^{i\sigma}{}_{,\sigma}\bar{h}^{kj}{}_{,i} \\ -2\bar{h}^{j\sigma}{}_{,\sigma}\bar{h}^{ki}{}_{,i} + (\bar{h}^{\alpha k,j} + \bar{h}^{\alpha j,k})\bar{h}_{\alpha\sigma}{}^{,\sigma} - 2\eta^{jk}(\bar{h}^{\alpha\sigma}{}_{,\sigma}\bar{h}^{i}{}_{\alpha})_{,i} - \eta^{jk}\mathcal{L}] \\ +\frac{1}{2}(\mathcal{T}^{k\alpha}h_{\alpha}{}^{j} + \mathcal{T}^{j\alpha}h_{\alpha}{}^{k}),\end{aligned} \quad (92)$$

$$\mathcal{L} = \frac{1}{32\pi G}[\frac{1}{2}\bar{h}_{\mu\nu,\lambda}\bar{h}^{\mu\nu,\lambda} - \frac{1}{4}\bar{h}_{,\lambda}\bar{h}^{,\lambda} - \bar{h}_{\mu\nu}{}^{,\mu}\bar{h}^{\lambda\nu}{}_{,\lambda}]. \quad (93)$$

It is easy to verify that total energy-momentum tensor consisting of (17) and (92) is conserved. The canonical part of MTW tensor in Hilbert gauge has the form

$$\begin{aligned}\overset{f}{T}{}^{\gamma\delta} = \frac{1}{32\pi G}\{\bar{h}^{\mu\nu,\delta}\bar{h}_{\mu\nu}{}^{,\gamma} - \frac{1}{2}\bar{h}^{,\delta}\bar{h}^{,\gamma} \\ -\eta^{\gamma\delta}[\frac{1}{2}\bar{h}_{\mu\nu,\lambda}\bar{h}^{\mu\nu,\lambda} - \frac{1}{4}\bar{h}^{,\lambda}\bar{h}_{,\lambda}]\} = \bar{T}^{\gamma\delta} - \frac{1}{2}\eta^{\gamma\delta}\bar{T}.\end{aligned} \quad (93a)$$

For Newtonian centers this expression coincides with Thirring's one, see eq.(6). As noted earlier the nonlocal part of $\overset{s}{T}{}^{\mu\nu}$ is zero for Newtonian centers.

For these centers from (92) and (9), (12) we have

$$\theta^{00} + \overset{int}{T}{}^{00} = \frac{1}{8\pi G}(\nabla\phi)^2 + 2\mu\phi,\ \mu = \mathcal{T}^{00}. \quad (94)$$

For this system the MTW Lagrangian (93) coincide with Thirring's one. The same is true for canonical energy-momentum tensors, see (6) and (86-87), but spin parts are different. It follows from (91) and (87) that for Newtonian centers MTW spin part contributes only to interaction tensor, while Thirring's spin part contributes also to pure field part, see (15).

We note now that in Hilbert gauge for static case (for Newtonian centers) the linearized Einstein equation can be written as

$$\nabla^2 h^W_{\mu\nu} = -\nabla^2 h^T_{\mu\nu} = -16\pi G\bar{T}_{\mu\nu}, \tag{95}$$

see (A12) below. As seen from (93a) for this system $\bar{T}_{00} = 0$, i.e. there is no contribution to $h_{00}$ from diagrams of Fig.2.

Comparing MTW and Thirring tensors in Newtonian approximation we see that addition of divergence terms to Lagrangian leads to the change in subdivision of energy density between purely field part and interaction part. Using (61-63) we obtain from (94) the effective gravitational energy density in Newtonian approximation given in (56).

Returning now to contributions from diagrams of Fig.2 we find that diagram (2a) leads to

$$ds^2 = (1+2\phi)dt - (1-2\phi-\phi^2)(d\vec{x})^2 - \phi^2\frac{x^i x^j}{r^2}dx^i dx^j. \tag{96}$$

Contribution from all 3 diagrams of Fig. 2 is represented in the expression

$$ds^2 = (1+2\phi)dt^2 - (1-2\phi-5\phi^2)(d\vec{x})^2 - 9\phi^2\frac{x^i x^l}{r^2}dx^i dx^j. \tag{97}$$

# 6 Discussion

We have assumed in this paper that in 3-graviton vertex each graviton interacts with gravitational energy-momentum tensor formed from two other gravitons. But in general relativity the 3-graviton vertex is given by cubic in $h_{\mu\nu}$ terms in function $G(x)$, see eq. (3), where only quadratic terms are written down. Correcting a misprint in [20] one finds that these terms are given by $Q_{\gamma\theta}$:

$$\begin{aligned}Q_{\gamma\theta} = \frac{1}{32\pi G}\{&\eta_{\gamma\theta}[h_{\alpha\beta,\lambda}h^{\lambda\beta,\alpha} - \frac{1}{2}h_{\alpha\beta,\lambda}h^{\alpha\beta,\lambda} - h_{\alpha\beta}{}^{,\alpha}h^{,\beta} + \frac{1}{2}h_{,\lambda}h^{,\lambda}] \\ &+h_{\alpha\beta,\gamma}h^{\alpha\beta}{}_{,\theta} - 2h_{\gamma\alpha,\beta}h_\theta{}^{\beta,\alpha} + 2h_{\gamma\alpha,\beta}h_\theta{}^{\alpha,\beta} + (h_{\gamma\alpha,\theta}+h_{\theta\alpha,\gamma})h^{,\alpha} \\ &-2h_{\gamma\theta,\alpha}h^{,\alpha} + (h_{\gamma\alpha}{}^{,\alpha}h_{,\theta} + h_{\theta\alpha}{}^{,\alpha}h_{,\gamma}) - h_{,\gamma}h_{,\theta} + 2h_{\gamma\theta,\alpha}h^{\alpha\beta}{}_{,\beta} \\ &-2(h_{\alpha\gamma,\beta}h^{\alpha\beta}{}_{,\theta} + h_{\theta\alpha,\beta}h^{\alpha\beta}{}_{,\gamma})\}.\end{aligned} \tag{98}$$

There are no reasons to expect that $Q_{\gamma\theta}$ is conserved energy-momentum tensor. Moreover it does not contain second derivatives and we know that the only conserved energy-momentum tensor with this property is Landau-Lifshitz

tensor [16]. Yet (98) does not coincide with LL tensor. This is confirmed also by the fact that (98) leads to general relativity result (73), while LL tensor leads to (63a).

We note now somewhat unexpected fact: the half sum of LL and MTW tensors reproduces general relativity result (76). On the other hand the half sum of Thirring tensor and MTW tensor gives

$$ds^2 = (1 + 2\phi + 2\phi^2) - (1 - 2\phi + 2\phi^2) + 2\phi^2 \frac{x^i x^j}{r^2} dx^i dx^j \qquad (99)$$

In such space-time a relativistic particle moves differently (in $G^2$ approximation) from what is expected according general relativity.

# 7 Concluding remarks

Though general covariance was not assumed, the gauge invariance is of course retained [2,10]. For this reason the weak gravitatinal waves in flat space are described as in general relativity. All considered tensors give the same energy-momentum tensor for the plane gravitational wave. There are no a priori reasons to believe that field-theoretical approach will give the same result as general relativity. It seems that there is still much to be done to synthesize the geometrical and field-theoretical aspects of gravitations.

I wish to thank V.I.Ritus, and I.V.Tyutin, for useful discussions and D.E.Ivanov for collaboration in evaluating the contribution from diagrams (2b), (2c).

# 8 Appendix

Using $\eta_{\mu\nu} = diag(-1, 1, 1, 1)$ we give here some details of calculating $h_{\mu\nu}$. We utilize the expression

$$h_{\mu\nu} = \int d^4x' D_{\mu\nu\rho\sigma}(x - x') t^{\rho\sigma}(x'), \qquad (A1)$$

where graviton propagator

$$D_{\mu\nu\rho\sigma}(x - x') = P_{\mu\nu\rho\sigma} D_+(x - x'), \qquad (A2)$$

$$D_+(x) = \frac{i}{(2\pi)^3} \int \frac{d^3p}{2p^0} \exp i(\vec{p}\vec{x} - p^0|t|), \qquad (A3)$$

$$P_{\mu\nu\rho\sigma} = \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}). \qquad (A4)$$

The polarization factor $P_{\mu\nu\rho\sigma}$ satisfies the relations

$$t^{\mu\nu} P_{\mu\nu\rho\sigma} = t_{\rho\sigma} - \frac{1}{2}\eta_{\rho\sigma} t \equiv \bar{t}_{\rho\sigma}, \quad P_{\mu\nu\rho\sigma} T^{\rho\sigma} = \bar{T}_{\mu\nu},$$

$$t^{\mu\nu}P_{\mu\nu\rho\sigma}T^{\rho\sigma} = t^{\mu\nu}T_{\mu\nu} - \frac{1}{2}tT = t^{\mu\nu}\bar{T}_{\mu\nu} = \bar{t}^{\mu\nu}T_{\mu\nu}. \qquad (A5)$$

The scalar factor $D_+(x)$ has the representation

$$D_+(x) = \frac{1}{4\pi}\delta_+(x^2) = \frac{i}{(2\pi)^2}\frac{1}{x^2 + i\epsilon}, \qquad (A6)$$

and possesses the property

$$\int d\tau D_+(\vec{x} - \vec{x}', \tau) = \frac{i}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{d\tau}{(\vec{x} - \vec{x}')^2 - \tau^2 + i\epsilon} = \frac{1}{4\pi|\vec{x} - \vec{x}'|}. \qquad (A7)$$

For spherically symmetric body we have to deal with integrals of the kind

$$\int d^4x' D_+(x - x')\rho(r') = \frac{1}{4\pi}\int\frac{d^3x'}{\sqrt{\vec{x}'^2 + \vec{x}^2 - 2\vec{x}\cdot\vec{x}'}}\rho(r')$$

$$= \frac{1}{r}\int_0^r dr' r'^2\rho(r') + \int_r^{\infty} dr' r'\rho(r'), \qquad (A8)$$

By the way it is seen from here that the derivative of Newtonian potential over $r$ is determined only by the mass inside sphere of radius $r$. Assuming in (A8) that $\rho = \frac{c}{r^4}$, we get

$$\frac{1}{r}\int_\delta^r dr' r'^2\rho(r') + \int_r^{\infty} dr' r'\rho(r') = c\left(\frac{1}{r\delta} - \frac{1}{2r^2}\right). \qquad (A9)$$

Hence the divergent part at small $r'$ appears only in the term, which is absorbed by mass renormalization.

So the source (13) generate the field

$$\bar{h}_{00} = 16\pi G\int d^4x' D_+(x - x')T_{00}(x') \Longrightarrow -\phi^2. \qquad (A10)$$

The arrow shows that the divergent part is included in mass renormalization. The r.h.s. of (A10) can be obtained also from the solution of wave equation derived from (A10) by action of the operator $\partial^2 = \nabla^2 - \frac{\partial^2}{\partial t^2}$ and taking into account that

$$-\partial^2 D_+(x - x') = \delta(x - x')\ , \quad \nabla^2\frac{1}{r^2} = \frac{2}{r^4}. \qquad (A11)$$

We note also that

$$h_{\mu\nu} = 16\pi G\int d^4x' D_+(x - x')\bar{T}_{\mu\nu}(x'). \qquad (A12)$$

Now we shall indicate how to obtain the contribution from diagram (2b). (The diagram (2c) contributes as much as (2b) and diagram of Fig.1 is irrelevant

to finding terms in $g_{\mu\nu}$.) First of all we remark that some terms in energy-momentum tensor are not symmetric in $h$ forming the tensor. Such terms ought to be symmetrized. For example the third term on the r.h.s. of (36) should be rewritten as

$$-2\varphi^{\alpha\beta}{}_{,\alpha\beta}\varphi_{\gamma\theta} \Rightarrow -\varphi^{\alpha\beta}{}_{,\alpha\beta}\varphi_{\gamma\theta} - \varphi_{\gamma\theta}\varphi^{\alpha\beta}{}_{,\alpha\beta}. \qquad (A13)$$

This is important because the left $\varphi$ will always be considered (for case of diagram (2b)) as "contained" in propagator and the right $\varphi$ as originating from Newtonian center. Moreover the second term on the r.h.s. of (A13) may be dropped as $\varphi$ coming from Newtonian center satisfies Hilbert condition (2). Rewritten in terms of $h$ (see (7) and (5)) the first term on the r.h.s. of (A13) has the form

$$-\varphi^{\alpha\beta}{}_{,\alpha\beta}\varphi_{\gamma\theta} = -\frac{1}{32\pi G}[h^{\alpha\beta}{}_{,\alpha\beta}h_{\gamma\theta} - \frac{1}{2}h_{,\alpha}{}^{\alpha}h_{\gamma\theta} - \frac{1}{2}\eta_{\gamma\theta}h^{\alpha\beta}{}_{,\alpha\beta}h + \frac{1}{4}\eta_{\gamma\theta}h_{,\alpha}{}^{\alpha}h] \qquad (A14)$$

Now we consider the contribution from the first term on the r.h.s. of (A14). Dropping temporarily the constant factor we have to evaluate the integral

$$\int d^4x'[D_{\mu\nu\alpha\beta}(x-x')]^{,\alpha\beta}h_{\gamma\theta}(x')h^{\gamma\theta}(x'). \qquad (A15)$$

The last $h$ represents the graviton of Newtonian center. This graviton interacts with the source, given by first term on the r.h.s. of (A14). Using (A2), (A4) and (69) we come to integrals

$$I_{\mu\nu} = -\int d^4x'[D_+(x-x')]_{,\mu\nu}\phi^2(x'), \qquad \int d^4x'[D_+(x-x')]_{,\alpha}{}^{\alpha}\phi^2(x') = -\phi^2(x). \qquad (A16)$$

The first equation in (A11) was used in the last equation in (A16). The first integral in (A16) is treated as follows. Integrating by parts we get

$$I_{\mu\nu} = \int d^4x'[D_+(x-x')]_{,\mu}2\phi(x')\phi_{,\nu}(x'). \qquad (A17)$$

As $\phi(x)$ is independent on $x^0$, $\nu = 0$ does not contribute. For the same reason $\mu = 0$ also does not contribute. Indeed for $\mu = 0$ we integrate over $x'^0$ and get the factor

$$D_+(x-x')|_{-\infty}^{+\infty} = 0,$$

see (A6). Thus we replace $\mu$ and $\nu$ by $i, j = 1, 2, 3$. Integrating (A17) over $x'^0$ we obtain

$$I_{ij} = \frac{G^2M^2}{4\pi}\int d^3x'\left(\frac{\partial}{\partial x'^j}\frac{1}{|\vec{x}-\vec{x}'|}\right)2\left(\frac{\partial}{\partial x'^i}\frac{1}{r'}\right)\frac{1}{r'} \qquad (A18)$$

Here we have used (A7) and expression for $\phi$, see text before eq. (10). Now writing

$$\frac{1}{r'}\frac{\partial}{\partial x'^i}\frac{1}{r'} = \frac{1}{2}\frac{\partial}{\partial x'^i}\frac{1}{r'^2} \qquad (A19)$$

and again integrating by parts we find

$$I_{\mu\nu} = -\frac{G^2M^2}{4\pi}\int d^3x' \frac{1}{|\vec{x}-\vec{x}'|}\frac{\partial^2}{\partial x'^i \partial x'^j}\frac{1}{r'^2}. \tag{A20}$$

Using relations

$$\left(\frac{1}{r^2}\right)_{,ij} = -\frac{2\delta_{ij}}{r^4} + \frac{8x^ix^j}{r^6} = \nabla^2\left(-\frac{2x^ix^j}{r^4} + \frac{\delta_{ij}}{r^2}\right) \tag{A21}$$

and again twice integrating by parts we get

$$I_{ij} = -\frac{G^2M^2}{4\pi}\int d^3x' \left(-2\frac{x'^ix'^j}{r'^4} + \frac{\delta_{ij}}{r'^2}\right)\nabla^2\frac{1}{|\vec{x}-\vec{x}'|} =$$

$$= G^2M^2\left(-2\frac{x^ix^j}{r^4} + \frac{\delta_{ij}}{r^2}\right) = \delta_{ij}\phi^2 - 2r^2\phi_{,i}\phi_{,j}. \tag{A22}$$

Finally restoring all factors we find that the contribution from the first term in (A14) is

$$-4\eta_{\mu\nu}\phi^2 + 8I_{\mu\nu}. \tag{A23}$$

The divergent integral

$$J = \int d^4x' D_+(x-x')(\nabla^2\phi(x'))\phi(x'),$$

appearing in some terms, is cancelled out in final result.

At last we show how to obtain the finite part of the integral

$$J_{ij} = \int d^4x' D_+(x-x')\phi_{,ij}(x')\phi(x') = G^2M^2\int\frac{d^3x'}{4\pi}\frac{1}{|\vec{x}-\vec{x}'|}\frac{3x'^ix'^j - \vec{x}'^2\delta_{ij}}{r'^6}. \tag{A24}$$

Utilizing relation

$$\nabla^2 I_{ij}(x) = \frac{r^2\delta_{ij} - 3x^ix^j}{r^6}G^2M^2,$$

and (cf.(42))

$$\nabla^2\left(\frac{3}{4}\frac{x^ix^j}{r^4} - \frac{\delta_{ij}}{4r^2}\right) = \frac{\delta_{ij}}{r^4} - \frac{3x^ix^j}{r^6}$$

we find that the finite part of $I_{ij}$ is

$$\frac{3}{4}r^2\phi_{,i}\phi_{,j} - \frac{\delta_{ij}}{4}\phi^2 \tag{A25}$$

The essential part of this Appendix machinery is checked up by obtaining the expression (73) starting from (98). Calculation of $g_{00}$ can be made by much easier method suggested by Schwinger, see [10] and [21]. This method uses more fully Hilbert condition and it is helpful for controlling some of our calculation. For example it is clear from this method that (A13) do not contribute to $h_{00}$.

Figure captions

Fig.1: The second rank tensor formed from matter energy-momentum tensor and graviton is a source for another graviton.

Fig.2: 3-graviton vertex. Short straight line serves only to distinguish the roles of participating gravitons: energy-momentum tensor is formed from two gravitons joining the straight line at its ends, this energy-momentum tensor serves as a source for graviton emerging from the middle of the straight line. Crosses represent external field sources.

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